# CORE MATHEMATICS (C) UNIT 2 TEST PAPER 4

1. In the triangle $ABC$, $AB = 2\sqrt{2}$ cm, $AC = 3.5$ cm and angle $BAC = \frac{\pi}{4}$ radians.

   Calculate the length of $BC$. [4]

2. Find the values of $a$ and $b$ for which $\sum_{r=0}^{n-1}(4r+2) = \sum_{s=1}^{n}(as+b)$. [5]

3. Given that $2\log_a x - \log_a 4 = 3$, where $a$ is a positive constant,

   (i) express $x$ in terms of $a$. [3]

   (ii) Find the value of $x$ when $a = 9$. [2]

4. Find the first four terms, in ascending powers of $x$, in the binomial expansion of

$$(3 - 4x)^5.$$ [6]

5. The table gives some values of $\sqrt{\log_{10} x}$.

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 |
|---|---|---|---|---|---|---|
| $\sqrt{\log_{10} x}$ | 0 | 0.420 | | 0.631 | | 0.738 |

   (i) Calculate, to 3 decimal places, the missing values in the table. [2]

   (ii) Using all six values, estimate the value of $\int_1^{3.5} \sqrt{\log_{10} x}\, dx$ to 2 decimal places. [4]

6. Find all the solutions in the interval $0 < x < 180$ of the equations

   (i) $\sin (x - 20)^\circ = \sin 80^\circ$, [3]

   (ii) $\tan 3x^\circ = -\sqrt{3}$. [4]

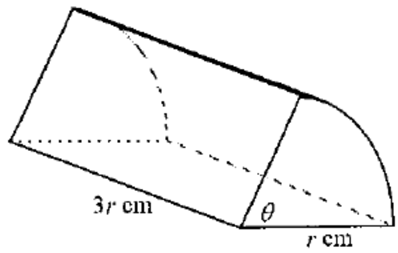
7. The first three terms of a geometric series are respectively $p, p^2, p^2 + 4$.

   (i) Show that the only possible real value of $p$ is 2. [5]

   (ii) If the $n$th term of this series is $T_n$ and the sum of the first $n$ terms is $S_n$, show that

$$S_n = 2(T_n - 1).$$ [4]

8. The diagram shows a prism whose cross-section is a sector of a circle of radius $r$ cm. The angle of the sector is $\theta$ radians. The length of the prism is $3r$ cm.




(i) Show that the total surface area of the prism is $2r^2(3 + 2\theta)$ cm$^2$. [4]

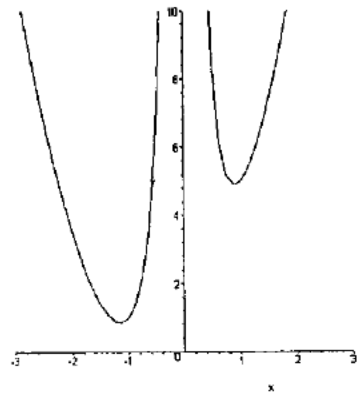
(ii) Find an expression in terms of $r$ and $\theta$ for the volume of the prism. [3]

If the volume of the prism has a fixed value of 36 cm$^3$,

(iii) express the surface area in terms of $r$ only. [3]

(iv) Show that the surface area is minimum when $r = 2$. [5]

9. The diagram shows the curve $C$ with equation $y = \mathrm{f}(x)$, where $x \neq 0$.




Given that $\mathrm{f}'(x) = 4x + 2 - \dfrac{4}{x^3}$, and that $C$ passes through the point $A$ (1, 5),

(i) find f($x$). [4]

(ii) Verify that $C$ also passes through $B$ (–1, 1). [2]

(iii) Show that the tangents to $C$ at $A$ and $B$ are parallel. [3]

(iv) Find the area between the curve $C$, the lines $x = -3$ and $x = -1$, and the $x$-axis. [6]

## CORE MATHS 2 (C) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1. $BC^2 = 8 + 12.25 - 14 = 6.25$ $\quad$ $BC = 2.5$ cm $\quad$ M1 A1 M1 A1 $\quad$ 4

2. $s = r + 1$, so $r = s - 1$ $\quad$ $as + b = 4(s-1) + 2$ $\quad$ $a = 4,\ b = -2$ $\quad$ M1 A1 M1 A1 A1 $\quad$ 5

3. (i) $\log_a (x^2/4) = 3$ $\quad$ $x^2 = 4a^3$ $\quad$ $x = 2a^{3/2}$ $\quad$ M1 A1 A1
   (ii) When $a = 9$, $x = 2 \times 27 = 54$ $\quad$ M1 A1 $\quad$ 5

4. $(3 - 4x)^5 = 3^5 + 5(3^4)(-4x) + 10(3^3)(-4x)^2 + 10(3^2)(-4x)^3 + \ldots$ $\quad$ M1 A1 A1
   $= 243 - 1620x + 4320x^2 - 5760x^3 + \ldots$ $\quad$ M1 A1 A1 $\quad$ 6

5. (i) (2, 0.549), (3, 0.691) $\quad$ B1 B1
   (ii) ½ (½) (0.738 + 2(2.291)) = 1.33 $\quad$ M1 M1 A1 A1 $\quad$ 6

6. (i) $x - 20 = 80, 100$ $\quad$ $x = 100, 120$ $\quad$ M1 A1 A1
   (ii) $3x = 120, 300$ $\quad$ $x = 40, 100, 160$ $\quad$ M1 A1 A1 A1 $\quad$ 7

7. (i) $(p^2)^2 = p(p^2 + 4)$ $\quad$ $p^4 - p^3 - 4p = 0$ $\quad$ $p(p-2)(p^2 + p + 2) = 0$ $\quad$ B1 M1 A1
   $p \neq 0$ (0, 0, 4 not a G.P.) and $p^2 + p + 2 = 0$ has no real roots, so $p = 2$ $\quad$ M1 A1
   (ii) $T_n = 2(2^{n-1}) = 2^n$ $\quad$ $S_n = 2(2^n - 1)/(2-1) = 2(T_n - 1)$ $\quad$ B1 M1 A1 A1 $\quad$ 9

8. (i) S.A. $= (r + r + r\theta)(3r) + 2(\tfrac{1}{2} r^2\theta) = 6r^2 + 4r^2\theta = 2r^2(3 + 2\theta)$ cm$^2$ $\quad$ M1 A1 M1 A1
   (ii) $V = 3r(\tfrac{1}{2} r^2\theta) = 3/2\ r^3\theta$ cm$^3$ $\quad$ M1 A1 A1
   (iii) $3/2\ r^3\theta = 36$ $\quad$ $\theta = 24/r^3$ $\quad$ S.A. $= 6r^2 + 96/r$ $\quad$ M1 A1 A1
   (iv) $d/dr$ (S.A.) $= 12r - 96/r^2 = 0$ when $r^3 = 8$ $\quad$ $r = 2$ $\quad$ M1 A1 A1
   Second derivative $= 12 + 192/r^3 > 0$, so minimum $\quad$ B2 $\quad$ 15

9. (i) Integrating, $f(x) = 2x^2 + 2x + 2/x^2 + c$ $\quad$ $5 = 6 + c$, so $c = -1$ $\quad$ M1 A1 M1 A1
   (ii) When $x = -1$, $2x^2 + 2x + 2/x^2 - 1 = 2 - 2 + 2 - 1 = 1$ $\quad$ M1 A1
   (iii) At $A$, gradient $= 4 + 2 - 4 = 2$ $\quad$ At $B$, gradient $= -4 + 2 + 4 = 2$ $\quad$ M1 A1 A1
   (iv) Area $= \int_{-3}^{-1} f(x)\,dx = \left[\frac{2x^3}{3} + x^2 - \frac{2}{x} - x\right]_{-3}^{-1} = \frac{10}{3} - \left[-\frac{16}{3}\right] = \frac{26}{3}$ $\quad$ M1 A2 M1 A1 A1

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